MORE SVD IN MATLAB

MATH 2271 - FALL 2023

(1) (Optional depending on time) First, if you want, you can check my work from Monday's class.

- Input the data matrix:

$$\text{data} = \begin{bmatrix} 88 & 81 & 92 & 86 & 81 & 82 \\ 92 & 79 & 93 & 84 & 84 & 78 \end{bmatrix}$$

- Try mean(data) and mean(data'). Which one do we want?
- Center the data using:

$$B = (\text{data}' - \text{mean}(\text{data}'))'$$

- There's many ways to visualize data in MATLAB. I don't know many of them but we can at least do: scatter$(B(1,:), B(2,:))$
- From the matrix A and compute the SVD.

$$A = (1/\sqrt{5}) * B'$$

  Use some code like [U,S,V] = svd(A)
- Look at the SVD (with rounding). Is it what I said it was?

(2) In the text file posted on Canvas are some 6×9 pixel gray scale images stored as matrices. We are going to use SVD to form a "composite" image which is the "best" combination of all of these images. I won't tell you what the images are of, you will have to wait for the SVD to figure out what the must have been of!

- Copy paste in the data matrices $A1, \ldots, A5$. They semi colons at the end of the line which means they will not display/print if you hit control enter. Don't display these!! That's cheating!!

- We will start with just A1, A2, A3, and A4: Form a "data matrix" by stretching out these matrices in to the columns of a big matrix:

$$D = [A1(:) \quad A2(:) \quad A3(:) \quad A4(:)]$$

- Center the data!

$$D_{avg} = (D - mean(D))$$

No transpose needed in this case for organization reasons (we already listed our data as the columns)

- We are not quite doing PCA so we will just compute the SVD of D_{avg}: [U,S,V] = svd(D_{avg})
- The first principal component is the first column of V. Let's give it a name: pcomp1 = V(:,1)
- Now we are ready to generate our composite image. It turns out, the way to combine all of these images into one best approximating composite image, is to take the principal components and multiply them by the original matrix D on the left. The resulting image will be sideways so we want to "reshape" the matrix too:

$$\text{composite} = \text{reshape}(D * \text{pcomp1}, 6, 9);$$

- Now we have MATLAB print the image! First, set colormap('gray'). Then run

$$\text{image}(\text{composite}, \text{CDataMapping} = \text{“scaled”})$$

- What do you see!?

(3) If you want to see the original images you can run, for $1 \leq i \leq 5$:

$$\text{image}(Ai, \text{CDataMapping} = \text{“scaled”})$$

From these you can see where the characteristics of the original 5 images were.

(4) Notice that A5 is bad data. Someone didn't follow the prompt. That's OK! SVD will weed them out. With only 5 samples, it doesn't work that well. But go back up to your original

D matrix and add some more artificial "good" images as well as $A5$. Your new D might look like

$$D = [A1(:) \quad A1(:) \quad A1(:) \quad A2(:) \quad A2(:) \quad A3(:) \quad A4(:) \quad A5(:)]$$

Run the code again. Even though we had some bad data (someone not following the prompt), the SVD still picks out the main theme!